Midterm exam Introduction to Logic (CS & MA)

Thursday 1 October 2015, 9 - 11 h.

- Write with a *blue or black pen* (so no pencil, no red pen).

- Write your *name and student number* on all pages of your work, and also the name of your *teaching assistant* and your *tutorial group* on the first page.

- With the regular exercises, you can earn 100 points. With the bonus exercise, you can earn additionally 10 points.
  The grade is the number of points you earned divided by 10, with a maximum of 10.

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1. (10 points)
   Translate the following sentences to *propositional logic*, where atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

   (a) The weather is not bad, provided that it does not rain and the sun shines.

   (b) The human body has enough oxygen only if the heart pumps blood around and the lungs work.

2. (10 points)
   Translate the following sentences to *first-order logic*. Do not forget to provide the translation key.

   (a) Amir and Ethel both vote for Greg unless Greg votes for Hans.

   (b) Jaime is happy but Karin is not, if Barcelona beats Ajax.

3. (30 points)
   Give formal proofs for the following inferences. Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.

   (a) $\neg P \rightarrow \bot$

   P

   (b) $(P \wedge Q \wedge R) \rightarrow S$

   $P \rightarrow (Q \rightarrow (R \rightarrow S))$

   (c) $\neg P(a) \wedge (a = b \vee P(b))$

   $a = b \leftrightarrow \neg P(b)$

4. (20 points)
Use *truth tables* to answer the next questions. Make the full truth table, and motivate your answer. Order the rows in the truth tables as follows:

P	Q	R	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

(a) Is $(P \lor Q \lor R) \land \neg Q \land (Q \leftrightarrow R)$ a contradiction?

(b) Are the following sentences *logically equivalent*? Indicate the spurious rows in the truth table.

   i. $((a = a \lor \mathsf{Tet}(a)) \land \neg\mathsf{Tet}(a)) \lor \neg(\mathsf{Small}(b) \to \neg(a = a))$

   ii. $(a = a \land \mathsf{Tet}(a)) \to \mathsf{Small}(b)$

5. (20 points)

(a) Provide a negation normal form (NNF) of the sentence $\neg(\neg\neg P \land (Q \lor \neg R))$.

(b) Provide a disjunction normal form (DNF) of the sentence $\neg(P \leftrightarrow (Q \to R))$.

Indicate in both cases the intermediate steps. You do not have to provide justifications for the steps.

6. (10 points)
Given the following three sets $A = \{0\}$, $B = \{0, 2, 3, 6\}$ and $C = \{1, 2, 3, 5\}$. For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

(a) $5 \in (A \cup B) \cap C$

(b) $A \cap B = \emptyset$

(c) $\emptyset \nsubseteq A$

(d) $A \subseteq B \setminus C$

(e) $A \in B$

(f) $(B \setminus C) \setminus A = \{6\}$

(g) $2 \notin (A \cap B) \cup (B \cap C)$

(h) $\emptyset \in A$

(i) $A \setminus B \subseteq C$

(j) $(B \cap C) \cup A \subseteq B$

7. (Bonus exercise: 10 points)
Give a formal proof for $((P \to Q) \to P) \to P$. Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.